Whence, $\phi = (\pi/2) - (\theta/2)$. Also

$$R = \frac{[1 + (dy/dx)^2]^{-\frac{3}{2}}}{d^2y/dx^2} = 4a \sin \theta/2$$

Hence, $P = v_0^2 \div (4a \sin \theta/2) - g \sin \theta/2$. But, since $y = \theta(1 - \cos \theta)$,

$$2 \sin^2 \frac{\theta}{2} = \frac{y}{a}.$$

Hence,

$$P = \frac{v_0^2 - 4ag \sin^2 \theta/2}{4a \sin \theta/2} = \frac{v_0^2 - 2gy}{4a \sin \theta/2},$$

But, at the point where the hound tumbled through, $y = y_0 = (v_0/2g)^2$. Hence, at that point, $P = 0$.

NUMBER THEORY.

212. Proposed by C. N. SCHMALL, New York City.

Given any positive integer N greater than 1; to prove that the sum of all the positive integers less than N and prime to N equals $\frac{1}{2}N \cdot \phi(N)$.

SOLUTION BY H. C. FEEMSTER, York College, Nebraska.

Let $N = a^h b^k c^l \cdots$, where $a, b, c, \cdots$ are primes. Then

$$\phi(N) = N \cdot \frac{a-1}{a} \cdot \frac{b-1}{b} \cdot \frac{c-1}{c} \cdot \cdots = N\left[1 - \Sigma\frac{1}{a} + \Sigma\frac{1}{ab} - \Sigma\frac{1}{abc} + \cdots\right],$$

the number of positive integers less than N and prime to N.

For every number $p < N/2$ and prime to N, there is a number $N - p > N/2$ and less than N and prime to N. Now there are $\phi(N)$ of these numbers or $\frac{1}{2}\phi(N)$ pairs of these numbers. But the sum of each pair is N, so the entire sum is $\frac{1}{2}N \cdot \phi(N)$, as required.

213. Proposed by R. D. CARMICHAEL, Indiana University.

Prove that no relatively prime integers x and y exist such that the difference of their fourth powers is a perfect cube.

SOLUTION BY ELIJAH SWIFT, University of Vermont.

We are to prove the non-existence of any equation $x^4 - y^4 = z^3$. There are two cases to consider: (1) *x and y both odd*; (2) *one even and one odd*.

In the second case $x^2 + y^2$ and $x^2 - y^2$ are prime to each other, since any common factor would divide their sum $2x^2$, and difference $2y^2$; but these have, by hypothesis, 2 as their only common factor, and 2 is not a factor of $x^2 + y^2$ because x is even and y odd, or vice versa.

Since the product of $x^2 + y^2$ and $x^2 - y^2$ is a perfect cube, each of the factors must be, say, $x^2 + y^2 = a^3$, $x^2 - y^2 = b^3$. Call $2xy = c$. Then $a^6 - b^6 = c^2$, which is impossible. (See Number Theory, problem 209 in the March, 1914, MONTHLY which denies the existence of such an equation.)